Lesson A - Natural Exponential Function and Natural Logarithm Functions

**Natural Exponential Function** In Lesson 21, we explored the world of logarithms in base 10. The natural logarithm has a base of “e”. “e” is approximately 2.718. The number e was discovered by a great 18th century mathematician named Euler. This famous irrational number is useful for determining rates of growth and decay. The derivation of e is beyond the scope of this course, but the expression \( \left(1 + \frac{1}{n}\right)^n \) produces results closer and closer to e, as the value substituted for “n” gets larger and larger.

The natural exponential function is \( f(x) = e^x \). Your scientific calculator should have a key for finding \( e^x \). You can also use the approximate value of e for computations. Below is the graph for \( f(x) = e^x \).

Rules of exponents apply to the exponential function. Study the examples of factoring.

*Example 1* \[
\frac{e^{2x} - e^x}{e^x} = \frac{e^x(e^x - 1)}{e^x} = e^x - 1
\]

Sometimes it is easier to factor by making a substitution. Let \( y = e^x \).

*Example 2* \[
\frac{4e^{2x} - 3e^x}{2e^x} \quad \text{Substituting yields} \quad \frac{4y^2 - 3y}{2y} = \frac{y(4y - 3)}{2y} = \frac{4y - 3}{2}
\]

Don’t forget to substitute back again. \[
\frac{4y - 3}{2} = \frac{4e^x - 3}{2}
\]

**Practice Problems**

Factor the following.

1) \( e^{2x} - 1 \)  
2) \( e^{2x} - e^x - 6 \)  
3) \( e^{2x} - e^x - 2 \)  
4) \( 2e^{2x} - 5e^x - 3 \)

**Solutions**

1) \( y^2 - 1 \)  
2) \( y^2 - y - 6 \)  
3) \( y^2 - y - 2 \)  
4) \( 2y^2 - 5y - 3 \)

\( (y - 1)(y + 1) \)  
\( (y - 3)(y + 2) \)  
\( (y - 2)(y + 1) \)  
\( (2y + 1)(y - 3) \)

\( (e^x - 1)(e^x + 1) \)  
\( (e^x - 3)(e^x + 2) \)  
\( (e^x - 2)(e^x + 1) \)  
\( (2e^x + 1)(e^x - 3) \)
Example 3  Suppose that the number of bacteria present in a culture is given by \( N(t) = 1000 e^{(-1)t} \).

How many bacteria would be in the culture when \( t = 4 \) hours?

\[
1000 e^{(-1)(4)} = 1000 e^{(-4)} = 1491.8 \text{ rounds to } 1492 \text{ bacteria}
\]

**Practice Problems**

1) How many bacteria would be in the culture above when \( t = 10 \) hours?

2) How long will it be before the number of bacteria reaches 10,000? Try to solve this by plugging in values for \( t \). We will learn how to solve this a different way soon.

**Solutions**

1) \( 1000 e^{(-1)(10)} = 1000 e^{(1)} = 2,718.3 \text{ rounds to } 2,718 \text{ bacteria} \)

2) try \( t = 15 \) hours

\[
1000 e^{(-1)(15)} = 1000 e^{(1.5)} = \text{ rounds to } 4482 \text{ bacteria}
\]

try \( t = 20 \) hours

\[
1000 e^{(-1)(20)} = 1000 e^{(2)} = 7389 \text{ bacteria}
\]

try \( t = 24 \) hours

\[
1000 e^{(-1)(24)} = 1000 e^{(2.4)} = 11,023 \text{ bacteria}
\]

You may have tried different values, but you should have determined that the time to reach 10,000 bacteria is somewhat less than 24 hours.

**Natural Logarithm Function**  The natural logarithm function is \( f(x) = \ln(x) \). In stands for natural log.

It is the inverse of the exponential function, which is \( f(x) = e^x \).

There are several properties and laws of the natural log function which you need to memorize. Use the directions that came with your scientific calculator to find and use the natural log key. (It may be marked LN.) Check the following relationships with your calculator, choosing any value you like for the variables.

1) \( \ln 1 = 0 \)
2) \( \ln e = 1 \)
3) \( \ln e^X = X \)
4) \( e^{\ln x} = x \text{ when } x \geq 0 \)
5) \( \ln xy = \ln x + \ln y \)
6) \( \ln x/y = \ln x - \ln y \)
7) \( \ln x^a = a \ln x \)

Example 4  Determine the value of \( \ln 2e \) in simplest terms.

\[
\ln 2e = \ln 2 + \ln e \quad \text{ Law #5 above }
\]

\[
= .69 + 1 \quad \text{ Use your calculator to find the natural log of 2.}
\]

\[
= 1.69 \quad \text{ You should know the natural log of e from #2 above.}
\]

\[
\ln 2e = 1.69 \quad \text{ Add to find the solution.}
\]
Practice Problems

Determine the value of each expression in simplest terms. Round your answers to the nearest hundredth.

1) \( \ln 4e^2 \)

2) \( \ln 8 - \ln 2 + \ln 5 \)

3) \( 4 \ln x + 2 \ln y \)

4) \( \ln \sqrt{5} \)

Solutions

1) \( \ln 4e^2 = \ln 4 + \ln e^2 \)
   \[ = 1.39 + 2 \ln e \]
   \[ = 1.39 + 2(1) \]
   \[ = 1.39 + 2 = 3.39 \]

2) \( \ln 8 - \ln 2 + \ln 5 = \ln 8/2 + \ln 5 \)
   \[ = \ln 4 + \ln 5 \]
   \[ = \ln 4 \cdot 5 \]
   \[ = \ln 20 = 3.0 \]

3) \( 4 \ln x + 2 \ln y = \ln x^4 + \ln y^2 \)
   \[ = \ln x^4 \cdot y^2 \]
   \[ = \ln x^4y^2 \]

4) \( \ln \sqrt{5} = \ln 5^{1/2} \)
   \[ = 1/2 \ln 5 \]
   \[ = 1/2 (1.609) = .80 \]

Example 8

Recall that on page 22-2, you were asked to find out how long it would be before the number of bacteria reached 10,000. Let's work that problem a different way using the natural logarithm function.

\[ 1000 e^{-0.1t} = 10,000 \]
\[ e^{-0.1t} = 10 \]
\[ \ln (e^{-0.1t}) = \ln (10) \]
Taking the natural logarithm of both sides.
\[ -0.1t = \ln 10 \]
\[ t = \frac{\ln 10}{-0.1} = 23 \text{ hours} \]
Exponential and logarithmic functions can be manipulated in algebraic equations. Remember that as long as we do the same thing to both sides of an equation, we do not change the value of the equation. In the examples below, find the natural log of each side in order to simplify exponents and put the equation in a form that is easier to manipulate.

Example 9

\[ 4^{x^2} = 9 \]
\[ \ln \left( 4^{x^2} \right) = \ln 9 \]
\[ x^2 \ln 4 = \ln 9 \]
\[ x^2 = \frac{\ln 9}{\ln 4} = \frac{2.197}{1.386} = 1.585 \]
\[ x = 1.26 \]

Example 10

\[ 4^{x+2} = 7^{2x-1} \]
\[ \ln \left( 4^{x+2} \right) = \ln \left( 7^{2x-1} \right) \]
\[ (x + 2) \ln 4 = (2x - 1) \ln 7 \]
\[ x \ln 4 + 2 \ln 4 = 2x \ln 7 - \ln 7 \]
\[ x \ln 4 - 2x \ln 7 = -2 \ln 4 - \ln 7 \]
\[ x \left( \ln 4 - 2 \ln 7 \right) = -2 \ln 4 - \ln 7 \]
\[ x = \frac{-2 \ln 4 - \ln 7}{\ln 4 - 2 \ln 7} = \frac{-2.77 - 1.946}{1.386 - 3.892} \]
\[ x = \frac{-4.716}{-2.506} = 1.88 \]
Lesson B - Limits

Limits in life are boundary points. Usually they denote the highest or top number allowable. Speed limits are the highest speed allowable which cars may travel without breaking the law. A supermarket limit of 2 quarts of strawberries on their store special means that you can buy up to 2 quarts per person and no more. In science an instrument might be accurate within 1/100th of an inch. In each case, the limit is a real number.

Mathematical limits are similar, but are not identical, to the limits we see in everyday life. In math, we refer to the limit of a function. A function’s limit may or may not exist. If the limit does exist, then the limit will be a real number and it will be unique.

Example 1  Consider the following sequence:

1, 1/2, 1/3, 1/4, 1/5, 1/6, ... 1/1000000, ....

As we look at these numbers, what is happening to them? They are getting smaller and smaller, right? How small will they get? Will they reach 0? No, but they will get so close to zero as to be indistinguishable. Therefore, the mathematical limit of these numbers is 0. In this case, the limit exists and is uniquely 0. It is not approaching any other number except 0.

Example 2  Now let’s look at another sequence:

1, 2, 3, 4, 5, 6, ... 10,000, ... 100,000,000 ....

What is happening to these numbers? They are getting larger and larger and are approaching infinity. Is there a limit? NO! If you say that the limit is 100,000,000,000,003, then I can always find a bigger number in the sequence. Therefore, the limit does not exist.

Example 3  In geometry, consider a circle which has a polygon inscribed inside. If you begin with a 3-sided polygon (triangle), and continue to inscribe polygons with larger and larger numbers of sides, what happens? Notice that the area of the inscribed polygon gets closer and closer to the area of the circle. Does the inscribed polygon become a circle? No, but it gets close enough as to be indistinguishable. Therefore, the limit exists and is uniquely the area of the circle. Historically, Archimedes used this exact reasoning to determine the area of circles before the discovery of pi.

Now let’s look at some functions on a graph, and see if we can understand limits more fully. Consider the following function.

Example 4  \( f(X) = |X + 1| \)
A limit in a graphical sense represents the anticipated height of a function. What would be the limit of \( f(X) \) as we allow our \( X \) values to get closer and closer to 0? We could approach 0 by using negative numbers or positive numbers. A chart below shows numbers which are approaching \( X = 0 \) from both perspectives.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( f(X) )</th>
<th>looking at positive values for ( X )</th>
<th>( X )</th>
<th>( f(X) )</th>
<th>looking at negative values for ( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td>-1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>1.5</td>
<td></td>
<td>-.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>.01</td>
<td>1.01</td>
<td></td>
<td>-.01</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>.00001</td>
<td>1.00001</td>
<td></td>
<td>-.00001</td>
<td>1.00001</td>
<td></td>
</tr>
</tbody>
</table>

Either way we look, we see that as the \( X \) value of this function approaches 0, the \( f(X) \) value or the height of the function will be approaching 1. Indeed, \( f(0) = 1 \)! Therefore, the limit exists and it is uniquely 1.

Example 5 \( f(X) = |X + 1| \) for all \( X \) except \( x = 0 \)

Graphically, we now have:

Now what is the limit as \( X \) approaches 0? It is still 1. Even though the value at \( X = 0 \) is undefined, we can get SO close to \( X = 0 \), and it still appears that \( f(X) \) is SO close to 1. How close is close enough? That is a discussion for Calculus.

Let’s try a few more:
Example 6 \( f(X) = X^2 \)

What is the limit of \( f(X) \) as \( X \) approaches -2? This one is simple. The graph will show the exact value of 4 when \( x = -2 \). You could look at values on both sides of -2, but your answer will be the same. The limit exists and it is uniquely equal to 4.
What is the limit of \( f(X) \) as \( X \) approaches 2? We will need a table here, because the value of 2 cannot be substituted into the function.

Therefore, the limit of \( f(X) = 3 \). Notice that the numerator of this function can be factored into \( (X + 1)(X - 2) \). Then, you can cancel the common factors, and you get \( X = 1 \). The graph looks like \( f(X) = X + 1 \), except that there is a hole at \( X = 2 \). In fact, if you substitute \( x = 2 \) into the new equation, \( f(X) = X + 1 \), you get \( f(2) = 2 + 1 = 3 \).

Now, let’s work with mathematical notation to accurately describe what we have done so far. The appropriate limit notation looks like the following:

\[
\lim_{X \to A} f(X) = B
\]

where “\( X \to A \)” means that \( X \) is approaching \( A \) and the value of the limit is \( B \).

In Example 7, we would express the limit as follows: \( \lim_{X \to A} \frac{X^2 - X - 2}{X - 2} = 3 \).
Example 8  What about more serious holes and gaps in a function? Refer to the graph below:

This type of function is commonly referred to as a “step function”. What is the limit as X approaches 2 of this step function?

<table>
<thead>
<tr>
<th>X</th>
<th>f(X)</th>
<th>X</th>
<th>f(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>2.1</td>
<td>2</td>
</tr>
<tr>
<td>1.9</td>
<td>1</td>
<td>2.001</td>
<td>2</td>
</tr>
<tr>
<td>1.9999</td>
<td>1</td>
<td>2.00001</td>
<td>2</td>
</tr>
</tbody>
</table>

As you can see, it depends on whether you approach X = 2 from the right or from the left, as to what you would expect the height of the function to be. From the right, it appears that the limit would be 2, and from the left, it appears that the limit would be 1. Remember that we said that in order for the limit to exist, that it had to be unique? Because of this, the limit as X approaches 2, does not exist. Written in appropriate limit notation, we get:

\[
\lim_{x \to 2} f(X) = \text{Does not exist (DNE)}
\]

For completeness, let’s write Examples 1, 2, 4, and 6 in proper limit notation.

Example 1:  \( \lim_{x \to \infty} \frac{1}{X} = 0 \)

Example 2:  \( \lim_{x \to \infty} = \text{Does not exist} \)

Example 4:  \( \lim_{x \to 0} |X + 1| \)

Example 6:  \( \lim_{x \to 2} X^2 = 4 \)

Let’s review:

1. Some limits exist and some do not.
2. If the limit exists, then it is a real number and it is unique.
3. The proper notation for a limit is:  \( \lim_{x \to A} f(X) = B \)
4. To evaluate the limit, first try substituting A into the function, (i.e., try f(A).)
5. If you cannot evaluate the limit by substituting, then try factoring.
6. If neither method works, then graph the function.
7. Finally, the limit is simply the anticipated height of f(X) when X = A.
Find the value of \( y \) for several values of \( x \) and graph each equation.

1) \( y = e^{2x} \)

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
0 & \\
1 & \\
-1 & \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
1 & \\
2 & \\
3 & \\
.5 & \\
\end{array}
\]

2) \( y = 2 \ln x \)

Factor.

3) \( \ln^2 x - \ln x - 2 \)

4) \( 2 \ln^3 x + 3 \ln^2 x \)

5) \( 3e^{2x} + 5e^x - 2 \)

6) \( \frac{e^{2x} - 1}{e^x - 1} \)

Solve for \( x \).

7) \( 2^x = 3^2x \)

8) \( 2^x = 4^{x-2} \)

9) \( e^{2x} = \ln 5 \)

Read the information given and answer the questions.

The number of bacteria present in a culture is given by \( N(t) = 2000 e^{3t} \).

10) How many bacteria will be in the culture when \( t = 2 \) hours?

11) How many hours would it take for the bacteria count to reach 100,000?
Find the value of \( y \) for several values of \( x \) and graph each equation.

1) \( y = e^{-2x} \)

\[
\begin{array}{c|c|c|c|c|c|c}
 x & -1 & 0 & 1 & 2 \\
\hline
 y & & & & \\
\end{array}
\]

2) \( y = \ln 2x \)

\[
\begin{array}{c|c|c|c|c|c|c}
 x & .125 & .25 & .5 & 1 & 2 & 3 \\
\hline
 y & & & & & & \\
\end{array}
\]

Factor.
3) \( \frac{2 \ln^3 x - 8 \ln x}{\ln x - 2} \)

4) \( 2 \ln^5 x + 5 \ln^4 x \)

5) \( 6e^{2x} + 4e^x - 2 \)

6) \( e^{2x} - \ln^2 x \)

Solve for \( x \).
7) \( e^x + 2^{-x} = 3 \)  Hint: First multiply both sides through by \( e^x \), then substitute \( y \) for \( e^x \).

8) \( 2^{5+x} = 3^{2-x} \)

9) \( e^{-3x} = \ln 4 \)

Read the information given and answer the questions.

The amount of money which has accumulated after \( t \) years at interest rate \( r \) under continuous compounding is \( M = Pe^{rt} \), where \( P \) = principle (original money invested), \( r \) = interest rate, and \( t \) = time in years.

10) If \( P = $600, r = .09, \) and \( t = 10 \), compute \( M \).

11) If $1000 was invested at 5% over 20 years, find \( M \).

12) How long would it take $1000 to become $1500 if it was invested at 6% interest?
Find the value of \( y \) for several values of \( x \) and graph each equation.

1) \( y = 2e^x \)

\[
\begin{array}{c|c}
 x & y \\
-3 & \ \ \\
-2 & \ \ \\
-1 & \ \ \\
0 & 1 \\
1 & \ \\
\end{array}
\]

2) \( y = \ln x^2 \)

Factor.

3) \( 3e^{2x} - 7e^x + 2 \)
4) \( 5e^{2x} - 14e^x - 3 \)
5) \( \frac{\ln^2 x - 4}{\ln x + 2} \)
6) \( 2 \ln^3 x + 6 \ln x \)

Solve for \( x \).

7) \( 3^{2x} = 4^{x-2} \)
8) \( e^x - e^{-x} = 4e^{-x} \)
9) \( \log_4 (\ln 5) = x \)

Read the information given and answer the questions.

The half-life of iodine-131 is 8 days. That is the time it takes for one half of the substance to decay or change into another kind of matter. The first question is done as an example.

10) How much of 6 grams of iodine-131 will remain after 10 days?

The formula needed is the decay formula: 

\[ Q(t) = I e^{-kt} \]

where \( I \) equals initial amount of the substance, \( t = \text{time} \), and \( k = \text{decay constant} \). The decay constant is different for different substances. For Iodine-131 it is approximately \( .087 \).

\[
\begin{align*}
Q(t) &= I e^{-kt} \\
Q(10) &= 6(e^{-0.087(10)}) \\
Q(10) &= 6(e^{-0.87}) \\
Q(10) &= 6(0.419) = 2.51 \text{ g remaining after 10 days}
\end{align*}
\]

11) How much iodine would be left after 12 days?

12) How many days would it take for only 1 gram of iodine to remain?
Find the value of \( y \) for several values of \( x \) and graph each equation.

1) \( y = e^x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
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<tbody>
<tr>
<td>-3</td>
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<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

2) \( y = \ln x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Factor.

3) \( \frac{5e^{2x} + 13e^x - 6}{e^x + 3} \)

4) \( 6e^{2x} - 12e^x \)

5) \( \frac{\ln^2 x - 1}{\ln^2 x + 2\ln x + 1} \)

6) \( 3\ln^3 x + 6\ln x \)

Solve for \( x \).

7) \( e^{2x} - 4 = 0 \)

8) \( \ln^2 x - 5\ln x + 6 = 0 \)

9) \( e^{3x} = \ln 2 \)

Read the information given and answer the questions.

The half-life of sodium-24 is 15 hours. When you are given the half-life of a substance, you can find the decay constant as follows. Start with the decay formula, and plug in the values you know.

\[
Q(t) = Se^{-kt}
\]

\[
\frac{1}{2} = 1e^{-k(15)}
\]

Set the size of the original sample to 1, and the size of the sample after 15 hours to 1/2.

\[
\ln \frac{1}{2} = \ln e^{-15k}
\]

\[
\ln 1/2 = -15k
\]

\[
\ln 1/2 = k = 0.046
\]

10) A 10 gram quantity of sodium-24 was studied. How much sodium-24 was left after 20 hours?

11) How many days would it take for only 2 grams of sodium-24 to remain?
Evaluate the following limits from the given graph.

1) \( \lim_{x \to 0} g(X) = \) __________

2) \( \lim_{x \to -2} r(X) = \) __________
\( \lim_{x \to 1} r(X) = \) __________

Evaluate the following limits by drawing a graph.

3) \( \lim_{x \to 1} |X - 2| = \) __________

4) \( \lim_{x \to \infty} \frac{1}{2x} = \) __________

Evaluate the following limits by factoring.

5) \( \lim_{x \to 2} \frac{X^2 + 3X - 10}{X - 2} = \) __________

6) \( \lim_{x \to 3} \frac{X^2 + X - 6}{X^2 + 2X - 3} = \) __________

Evaluate the following limits using any appropriate method.

7) \( \lim_{x \to 1} 2X^2 - 6 = \) __________

8) \( \lim_{x \to \infty} X^2 + 2X = \) __________

9) \( \lim_{x \to \pi/2} \frac{\cos X}{\sin^2 X} = \) __________

10) \( \lim_{x \to 0^+} \frac{\cot^2 \theta - \csc^2 \theta}{\sec^2 \theta} = \) __________
    Hint: Simplify and then evaluate.
Evaluate the following limits from the given graph.

1) \( w(X) \)

\[
\lim_{{x \to 2}} w(X) = \underline{\phantom{00000}}
\]

2) \( q(X) \)

\[
\lim_{{x \to 1}} q(X) = \underline{\phantom{00000}}
\]

\[
\lim_{{x \to 2}} q(X) = \underline{\phantom{00000}}
\]

Evaluate the following limits by drawing a graph.

3) \( \lim_{{x \to \infty}} 1/x \)

4) \( \lim_{{\theta \to \pi/2}} \sin \theta \)

Evaluate the following limits by factoring.

5) \( \lim_{{x \to 3}} \frac{2x^2 - 6}{x - 3} \)

6) \( \lim_{{x \to 4}} \frac{x^2 + 9x - 20}{x - 4} \)

Evaluate the following limits using any appropriate method.

7) \( \lim_{{y \to 10}} \log y \)

8) \( \lim_{{x \to 0}} \frac{2e^x(e^x - 1)}{e^{2x} - e^x} \quad \text{Hint: Factor, then evaluate.} \)

9) \( \lim_{{x \to 2}} \frac{1}{x^2 - 1} \)

10) \( \lim_{{x \to 3}} \frac{x - 3}{3 - X} \)
Evaluate the following limits from the given graph.

1) \( s(X) \)
   \[
   \lim_{{x \to 2}} s(X) = 
   \]

2) \( p(X) \)
   \[
   \lim_{{x \to \infty}} p(X) = 
   \]

Evaluate the following limits by drawing a graph.

3) \( \lim_{{x \to 1}} \frac{X^2 - X}{x - 1} \)

4) \( \lim_{{\theta \to \pi/2}} \cos \theta \)

Evaluate the following limits by factoring.

5) \( \lim_{{x \to 1}} \frac{X^2 - 1}{x - 1} \)

6) \( \lim_{{x \to 0}} \frac{3X - 7X^2}{X} \)

Evaluate the following limits using any appropriate method.

7) \( \lim_{{\theta \to 90^\circ}} \frac{\sec^2 \theta - 1}{3\sec^2 \theta} \)

8) \( \lim_{{x \to \infty}} \pi^x \)

9) \( \lim_{{x \to 2}} \frac{X^2 - 7X + 10}{X^2 - 4} \)

10) \( \lim_{{x \to 0}} \frac{(X+3)^3 - 27}{X} \)  Hint: Use Pascal's triangle to expand \((X+3)^3\).
Evaluate the following limits from the given graph.

1) \[ \lim_{x \to 2} v(X) = \ldots \]

2) \[ \lim_{x \to \pi} s(X) = \ldots \]

Evaluate the following limits by drawing a graph.

3) \[ \lim_{x \to \infty} e^x = \ldots \]

4) \[ \lim_{\theta \to 180^\circ} 2 \tan \theta = \ldots \]

Evaluate the following limits by factoring.

5) \[ \lim_{x \to 3} \frac{3x^3 - 27x}{x - 3} = \ldots \]

6) \[ \lim_{x \to 4} \frac{4 - x}{x^2 - 16} = \ldots \]

Evaluate the following limits using any appropriate method.

7) \[ \lim_{x \to 2} \frac{4x}{\sqrt{7 + x}} = \ldots \]

8) \[ \lim_{x \to \pi/2} \frac{\cos x}{\cot x} = \ldots \]

9) \[ \lim_{\theta \to 90^\circ} \frac{\sin \theta}{1 - \cos \theta} \cdot \frac{1}{\csc \theta + \cot \theta} \]

Hint: Multiply through by the conjugate for \(1 - \cos \theta\).

10) \[ \lim_{x \to 0} \frac{e^{2x} + e^x - 2}{e^x - 1} \]

Hint: Factor the numerator.
1) The natural logarithm function, $\ln x$, is the inverse of
   A) $\sin X$  
   B) $\cos X$  
   C) $e^x$  
   D) $\log x$

2) The graph of the $e$ function lies in which quadrants?
   A) I only  
   B) II only  
   C) I and II  
   D) III only

3) The value of $e$ is approximately
   A) 27  
   B) 2.7  
   C) 270  
   D) .27

4) The factorization of $\ln^2 x - 1$ is
   A) $(\ln x - 1)(\ln x - 1)$  
   B) $(\ln x - 1)(\ln x + 1)$  
   C) $2\ln x(-x)$  
   D) $\log x 2A \div \log x B$

5) The factorization of $2e^{2x} + 5e^x - 12$ is
   A) $(2e^{2x} - 3)(e^x + 4)$  
   B) $(2e^x + 3)(e^x - 4)$  
   C) $(2e^x - 6)(e^x + 2)$  
   D) $(2e^x - 3)(e^x + 4)$

6) If $e^{2x} = 8$, then $x =$
   A) 1.03  
   B) 2.08  
   C) .69  
   D) .35

7) If $e^{2+x} = e^{\ln 3+1}$, then $x =$
   A) .1  
   B) 1.1  
   C) 4.1  
   D) 3.1

8) If $\ln(4e^x) = -6$, then $x =$
   A) 1.4  
   B) 2.4  
   C) 7.4  
   D) 4.6

9) If the number of bacteria present is given by $N(t) = 5000e^{-3t}$, how many bacteria will be present when $t = 5$ hours?
   A) 6,749  
   B) 22,408  
   C) 16,600  
   D) $1.6 \times 10^{10}$

10) Use the formula from #9 to find how long it will be until there are 50,000 or more bacteria.
   A) nearly 10 hours  
   B) nearly 16 hours  
   C) nearly 17 hours  
   D) nearly 8 hours

11) A triangle has side $c = 28$, $m\angle A = 34^\circ$ and $m\angle C = 72^\circ$. Use the law of sines to find the length of side $a$.
   A) 13.2  
   B) 16.5  
   C) 52.8  
   D) 28

12) What are the reference angle and $\tan$ for $405^\circ$?
   A) $45^\circ, \sqrt{2}$  
   B) $45^\circ, 1$  
   C) $60^\circ, \sqrt{3}$  
   D) $315^\circ, 1$

13) $\cos(90^\circ - \theta) = \frac{1}{\csc \theta}$
   A) $\cos^2 \theta$  
   B) $\tan \theta$  
   C) $\sin \theta \cos \theta$  
   D) $\sin^2 \theta$

14) A triangle has sides $a = 8$, $b = 4$ and $c = 11$. Use the law of cosines to solve for angle $A$.
   A) $90.7^\circ$  
   B) $85.4^\circ$  
   C) $33.9^\circ$  
   D) $.83^\circ$

15) Change $r = \frac{6}{3 \cos \theta - \sin \theta}$ to rectangular form.
   A) $3X - Y = 6$  
   B) $X - Y = 2$  
   C) $X - 3Y = 6$  
   D) $3X^2 - Y^2 = 2$
1) Which of the following methods can be used to compute a limit?
   I) graphing  II) factoring
   III) the Pythagorean theorem  IV) quadratic formula
   A) III and IV  B) I only
   C) I and II only  D) I, II and III

2) Which of the following statements is not true?
   A) Some limits exist.  B) A limit is a complex number.
   C) A limit is the anticipated height of a function.  D) Limits are unique.

3) \( \lim_{x \to 2\pi} \cos \theta = \)
   A) 0  B) 1  C) -1  D) does not exist

4) \( \lim_{x \to 5} \frac{-x^2 + 3x + 10}{x - 5} = \)
   A) 7  B) does not exist  C) -3  D) -7

5) The graph is \( Y = \cot \theta \).

6) \( \lim_{x \to 0} \frac{e^{-2x} + 2e^{-x}}{e^{-x}} = \)
   A) 0  B) 1  C) 2  D) does not exist

7) \( \lim_{\theta \to 0^\circ} \frac{\cos^2 \theta - 1}{\sin \theta (\sin \theta - 1)} = \)
   A) 0  B) 1  C) -1  D) does not exist

8) \( \lim_{x \to 1} \frac{2 \ln^2 x + 3 \ln x}{\ln x} = \)
   A) 3  B) -3  C) 0  D) does not exist

9) \( \lim_{x \to 4} \frac{1}{x^4 - \sqrt{x}} = \)
   A) 1/254  B) 254  C) 0  D) does not exist

10) \( \lim_{x \to \infty} \frac{-1}{2x^2} + 7 = \)
    A) 7  B) -2  C) 0  D) does not exist

11) If \( |x + 1| < -2 \), then the solutions for \( x \) are
    A) \( x > 1 \)  B) \( x < -3 \)
    C) \( x > -3 \)  D) no solutions

12) If \( \sqrt{x + 1} < 7 \), then the solutions for \( x \) are
    A) \( x < 48 \)  B) \( x > 48 \)
    C) \( x < 49 \)  D) \( x < 6 \)

13) \( -\sqrt[3]{B^3} = \)
    A) \( \frac{3}{B^2} \)  B) \( B \)
    C) \( \frac{3}{B^4} \)  D) \( B^\frac{1}{3} \)

14) \( \frac{3(1 - \cos^2 \theta)}{\cos(90^\circ - \theta)} \) is equal to which of the following when \( \theta = 90^\circ \)?
    A) 0  B) 3  C) 1  D) -3

15) \( \frac{B^2B^{-3}C^4}{C^2B} = \)
    A) \( C^6/B^2 \)  B) \( B^2/C^6 \)
    C) \( C^4B^{-2} \)  D) \( C^{-4}B^2 \)
1) \( y = e^{2x} \)

2) \( y = 2 \ln x \)

3) \( \ln^2 x - \ln x - 2 \)
   \( Y^2 - Y - 2 = \)
   \( (Y - 2)(Y + 1) = \)
   \( (\ln x - 2)(\ln x + 1) = \)

4) \( 2 \ln^3 x + 3 \ln^2 x \)
   \( 2Y^3 + 3Y^2 = \)
   \( Y^2(2Y + 3) = \)
   \( \ln^2 x(2 \ln x + 3) = \)

5) \( 3e^{2x} + 5e^x - 2 \)
   \( 3Y^2 + 5Y - 2 = \)
   \( (3Y - 1)(Y + 2) = \)
   \( (3e^x - 1)(e^x + 2) = \)

6) \( \frac{Y^2 - 1}{Y - 1} \)
   \( \frac{(Y + 1)(Y - 1)}{(Y - 1)} = \)
   \( Y + 1 = e^x + 1 \)

7) \( 2^{5x} = 3^{2x} \)
   \( \ln (2^{5x}) = \ln (3^{2x}) \)
   \( 5X \ln 2 = 2X \ln 3 \)
   \( 5X \ln 2 - 2X \ln 3 = 0 \)
   \( X(5 \ln 2 - 2 \ln 3) = 0 \)
   \( X = 0 \)

8) \( 2^x = 4^{x-2} \)
   \( \ln 2^x = \ln 4^{x-2} \)
   \( X = \ln 4 \)
   \( \frac{X}{X - 2} = \frac{\ln 4}{\ln 2} \)
   \( X = 2(X - 2) \)
   \( X = 2X - 4 \)
   \( -X = -4 \)
   \( X = 4 \)

9) \( e^{2x} = \ln 5 \)
   \( \ln (e^{2x}) = \ln (\ln 5) \)
   \( 2X = \ln (\ln 5) \)
   \( X = \frac{\ln (\ln 5)}{2} = .24 \)

10) \( N_t = 2000e^{6} = 3644 \)

11) \( 100,000 = 2,000e^{3t} \)
    \( 50 = 3^{3t} \)
    \( \ln 50 = .3t \)
    \( \ln 50 = t \)
    \( .3t = t \)
    \( t = 13 \)
1) $y = e^{-2x}$

<table>
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<tr>
<td>-1</td>
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<tr>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>.1</td>
</tr>
<tr>
<td>2</td>
<td>.02</td>
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2) $y = \ln 2x$

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<th>y</th>
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<td>-.7</td>
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<tr>
<td>.5</td>
<td>0</td>
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<td>1</td>
<td>7</td>
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<tr>
<td>2</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

3) \[
\frac{2Y^3 - 8Y}{Y - 2} = \frac{2Y(Y^2 - 4)}{Y - 2} = \frac{2Y(Y + 2)(Y - 2)}{Y - 2} = 2\ln X(\ln X + 2)
\]

4) \[2Y^5 + 5Y^4 = Y^4(2Y + 5) = \ln^4 X(2\ln X + 5)\]

5) \[6Y^2 + 4Y - 2 = 2(3Y^2 + 2Y - 1) = 2(3Y - 1)(Y + 1) = 2(3e^X - 1)(e^X + 1)\]

6) \[e^{2X} - \ln^2 X = (e^X - \ln X)(e^X + \ln X)\] (difference of 2 squares)

7) \[e^X + 2^{-X} = 3\]
\[e^X + 2e^0 = 3X\]
\[e^X - 3e^X + 2 = 0\]
\[Y^2 - 3Y + 2 = 0\]
\[(Y - 1)(Y - 2) = 0\]
\[(e^X - 1)(e^X - 2) = 0\]

8) \[2^{5+X} = 3^{2-X}\]
\[5\ln 2 + X\ln 2 = 2\ln 3 - X\ln 3\]
\[X\ln 2 + X\ln 3 = 2\ln 3 - 5\ln 2\]
\[X(\ln 2 + \ln 3) = 2\ln 3 - 5\ln 2\]
\[X = \frac{2\ln 3 - 5\ln 2}{\ln 2 + \ln 3} \approx -.73\]

9) \[e^{-3X} = \ln 4\]
\[\ln(e^{-3X}) = \ln(\ln 4)\]
\[-3X = \ln(\ln 4)\]
\[X = \frac{\ln(\ln 4)}{-3} \approx .11\]

10) $M = 600e^{(.09)(10)}$
\[M = 600e^9\]
\[M = 1,475.76\]

11) $M = 1000e^{(.05)(20)}$
\[M = 1000e^1\]
\[M = 2,718.28\]

12) $M = Pe^{rt}$
\[1500 = 1000e^{.06t}\]
\[1.5 = e^{.06t}\]
\[\ln 1.5 = \ln e^{.06t}\]
\[\ln 1.5 = .06t\]
\[t = \frac{\ln 1.5}{.06} = 6.76\] or about $6\frac{3}{4}$ years
1) \( y = 2e^x \)

2) \( y = \ln x^2 \)

3) \[ 3Y^2 - 7Y + 2 = \frac{(3Y - 1)(Y - 2)}{(3e^x - 1)(e^x - 2)} \]

4) \[ 5Y^2 - 14Y - 3 = \frac{(5Y + 1)(Y - 3)}{(5e^x + 1)(e^x - 3)} \]

5) \[ \frac{Y^2 - 4}{Y + 2} = \frac{Y + 2}{(Y + 2)(Y - 2)} \]

6) \[ 2Y^3 + 6Y = 2Y(Y^2 + 3) = 2 \ln X \left( \ln^2 X + 3 \right) \]

7) \[ 3^{2x} = 4^{x-2} \]

8) \[ e^x - e^{-X} = 4e^{-x} \]

9) \[ \log_4 (\ln 5) = x \]

10) done

11) \[ Q(t) = Ie^{-kt} \]

\[ Q(12) = 6e^{-12(0.087)} = 6e^{-1.044} = 2.11 \text{ g} \]

12) \[ Q(t) = Ie^{-kt} \]

\[ \frac{1}{6} = e^{-0.087(t)} \]

\[ \ln(1/6) = \ln e^{-0.087(t)} \]

\[ \ln(1/6) = -0.087(t) \]

\[ \frac{\ln(1/6)}{-0.087} = t \approx 20.6 \text{ days} \]
1) \[ y = e^x + 2 \]

2) \[ y = \ln x \]

3) \[
\frac{5Y^2 + 13Y - 6}{Y + 3} = \frac{(Y + 3)(5Y - 2)}{Y + 3} = 5Y - 2 = 5e^x - 2
\]

4) \[
6Y^2 - 12Y = 6Y(Y - 2) = 6e^x(e^x - 2)
\]

5) \[
\frac{Y^2 - 1}{Y^2 + 2X + 1} = \frac{(Y + 1)(Y - 1)}{Y + 1} = \frac{Y - 1}{Y + 1} = \ln X - 1
\]

6) \[
3Y^3 + 6Y = 3Y(Y^2 + 2) = 3 \ln X(\ln^2 X + 2)
\]

7) \[
Y^2 - 4 = 0
\]
\[
(Y + 2)(Y - 2) = 0
\]
\[
Y = 2 \quad Y = -2
\]
\[
e^X = 2 \quad e^X = -2
\]
\[
X = \ln 2 \quad X = \ln(-2)
\]
\[
X \approx 0.69 \quad X \text{ is undefined}
\]

8) \[
Y^2 - 5Y + 6 = 0
\]
\[
(Y - 2)(Y - 3) = 0
\]
\[
Y = 2 \quad Y = 3
\]
\[
\ln X = 2 \quad \ln X = 3
\]
\[
X = e^2 \quad X = e^3
\]
\[
X \approx 7.39 \quad X \approx 20.09
\]

9) \[
e^{3X} = \ln 2
\]
\[
\ln e^{3X} = \ln(\ln 2)
\]
\[
3X = \ln(\ln 2)
\]
\[
X = \frac{\ln(\ln 2)}{3} = -0.12
\]

10) \[
Q(t) = Se^{-kt}
\]
\[
= 10e^{-0.046(20)}
\]
\[
= 10e^{-0.92}
\]
\[
\approx 10(0.4) = 4 \text{ g}
\]

11) \[
Q(t) = Se^{-kt}
\]
\[
2 = 10e^{-0.046(t)}
\]
\[
\frac{1}{5} = e^{-0.046t}
\]
\[
\ln(1/5) = -0.046t
\]
\[
\ln(1/5) = t \approx 35 \text{ hours}
\]
\[
35 \div 24 = 1.4 \text{ days}
\]
1) \( \lim_{x \to 0} g(x) = 0 \)
   From the left and from the right, the values for \( g(x) \) get closer to 0 as \( x \) approaches 0.

2) \( \lim_{x \to -2} r(x) = \text{DNE} \) (does not exist)
   From the left the limit = -2.
   From the right the limit = 0

3) \( \lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 5)}{x - 2} = \lim_{x \to 2} (x + 5) = 2 + 5 = 7 \)

4) \( \lim_{x \to -1} |x - 2| = |1 - 2| = 1 \)

5) \( \lim_{x \to 0} \frac{x^2 + 3x - 10}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 5)}{x - 2} = \lim_{x \to 2} (x + 5) = 2 + 5 = 7 \)

6) \( \lim_{x \to -3} \frac{x^2 + x - 6}{x^2 + 2x - 3} = \lim_{x \to -3} \frac{(x - 2)(x + 3)}{(x + 3)(x - 1)} = \lim_{x \to -3} \frac{x - 2}{x - 1} = \frac{-3 - 2}{-3 - 1} = \frac{5}{4} = \frac{5}{4} \)

7) \( \lim_{x \to 1} 2x^2 - 6 = 2(1)^2 - 6 = -4 \)

8) \( \lim_{x \to \infty} x^2 + 2x \) DNE These numbers will grow without bound.

9) \( \lim_{x \to \pi/2} \frac{\cos x}{\sin^2 x} = \frac{\cos \pi/2}{\sin^2 \pi/2} = 0 = 0 \)

10) \( \lim_{x \to 0^0} \frac{\cos^2 \theta - \csc^2 \theta}{\sec^2 \theta} = \lim_{x \to 0^0} \frac{\cos^2 \theta - \frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta}} = \lim_{x \to 0^0} \frac{\sin^2 \theta - 1}{\sin^2 \theta} \times \frac{\cos^2 \theta}{1} = \lim_{x \to 0^0} \frac{\sin^2 \theta}{\sin^2 \theta} \times \frac{\cos^2 \theta}{1} = \lim_{x \to 0^0} -\sin^2 \theta \times \cos^2 \theta = \lim_{x \to 0^0} -\cos^2 \theta = -\cos^2 0^0 = -1 \)

11) \( \lim_{x \to 2} W(x) = 0 \)

12) \( \lim_{x \to -1} q(x) = \text{DNE} \)
   \( \lim_{x \to 2} q(x) = 3 \)

13) \( \lim_{x \to \infty} 1 \)

14) \( \lim_{y \to 10} \log y = \log 10 = 1 \)

15) \( \lim_{x \to 0} \frac{2e^x}{e^x - 1} = \lim_{x \to 0} \frac{2e^x}{e^x - 1} = 2 \)

16) \( \lim_{x \to -3} \frac{x - 3}{x - 4} = \lim_{x \to -3} \frac{-x - 3}{x - 4} = \lim_{x \to -3} \frac{-x - 3}{x - 4} = \lim_{x \to -3} (x - 5) = -(4 - 5) = 1 \)

17) \( \lim_{x \to 0} \frac{1}{x^2 - 1} = \frac{1}{2^2 - 1} = \frac{1}{3} \)

18) \( \lim_{x \to 3} \frac{3 - x}{3 - x} = \frac{3 - 3}{3 - 3} = -1 \)
1) \( \lim_{x \to 2} s(x) = \text{DNE} \)
2) \( \lim_{x \to \infty} p(x) = 0 \)
3) \( \lim_{x \to 1} \frac{x^2 - x}{x - 1} = (x - 1) \frac{x}{x - 1} = \lim_{x \to 1} x = 1 \)
4) \( \lim_{\theta \to \pi/2} \cos \theta = 0 \)
5) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} (x - 1)(x + 1) = \lim_{x \to 1} x + 1 = 2 \)
6) \( \lim_{x \to 0} \frac{3x^2 - 7x^2}{x} = \lim_{x \to 0} \frac{x(3 - 7x)}{x} = 3 - 7(0) = 3 \)
7) \( \lim_{\theta \to 90^\circ} \frac{\sec^2 \theta - 1}{3 \sec^2 \theta} = \lim_{\theta \to 90^\circ} \frac{\tan^2 \theta}{3 \sec^2 \theta} = \frac{\sin^2 \theta}{3 \left( \frac{1}{\cos^2 \theta} \right)} = \lim_{\theta \to 90^\circ} \frac{\sin^2 \theta}{3} = \sin^2 90^\circ = \frac{1}{3} \)
8) \( \lim_{x \to \infty} \pi^x = \text{DNE} \)
9) \( \lim_{x \to 2} \frac{x^2 - 7x + 10}{x^2 - 4} = \lim_{x \to 2} \frac{(x - 2)(x - 5)}{(x + 2)(x - 2)} = \lim_{x \to 2} \frac{x - 5}{x + 2} = \frac{2 - 5}{2 + 2} = -\frac{3}{4} \)
10) \( \lim_{x \to 0} \frac{(x + 3)^3 - 27}{x} = \lim_{x \to 0} \frac{x^3 + 9x^2 + 27x + 27 - 27}{x} = \lim_{x \to 0} \frac{x^3 + 9x^2 + 27x}{x} = \lim_{x \to 0} x^2 + 9x + 27 = 27 \)
11) \( \lim_{x \to \infty} V(x) = \text{DNE} \)
12) \( \lim_{x \to 4} V(x) = 2 \)
13) \( \lim_{x \to \pi} s(x) = 0 \)
14) \( \lim_{x \to \infty} = \text{DNE} \)
15) \( \lim_{x \to 3} \frac{3x^2 - 27x}{x - 3} = \lim_{x \to 3} \frac{3x(x^2 - 9)}{x - 3} = \lim_{x \to 3} 3x(x + 3) = 3(3)(3 + 3) = 54 \)
16) \( \lim_{x \to 4} \frac{4x - x}{x + 4} = \lim_{x \to 4} \frac{x(4 - 1)}{x + 4} = \lim_{x \to 4} \frac{4 - 1}{x + 4} = \frac{4 - 1}{8} = \frac{3}{8} \)
17) \( \lim_{x \to \pi/2} \cos x = \lim_{x \to \pi/2} \cos x = \lim_{x \to \pi/2} \frac{\cos x}{\sin x} = \lim_{x \to \pi/2} \frac{\sin x}{\sin x} = \lim_{x \to \pi/2} 1 = 1 \)
18) \( \lim_{x \to 90^\circ} \frac{\sin \theta}{1 - \cos \theta} = \frac{1}{\csc \theta + \cot \theta} \)
19) \( \lim_{x \to 90^\circ} \frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\csc \theta + \cot \theta} \)
20) \( \lim_{x \to 90^\circ} \frac{\sin \theta + \sin \theta \cos \theta}{1 - \cos^2 \theta} = \frac{1}{\csc \theta + \cot \theta} \)
21) \( \lim_{x \to 90^\circ} \frac{\csc \theta + \cot \theta}{\sin \theta + \sin \theta \cos \theta} = \frac{1}{\sin \theta + \sin \theta \cos \theta} \)
22) \( \lim_{x \to 90^\circ} \frac{e^{2x} + e^x - 2}{e^x - 1} = \lim_{x \to 0} \frac{(e^x - 1)(e^x + 2)}{e^x - 1} = \lim_{x \to 0} e^x + 2 = 1 + 2 = 3 \)
Test A

1) C
2) C
3) B

4) B: $y = \ln x$
   \[ y^2 - 1 = (y + 1)(y - 1) = (\ln x + 1)(\ln x - 1) \]

5) D: $y = e^x$
   \[ 2y^2 + 5y - 12 = (2y - 3)(y + 4) \]

6) A: $e^{2x} = 8$
   \[ 2x = \ln 8 \]
   \[ x = \frac{\ln 8}{2} = 1.03 \]

7) A: $e^{2 + x} = \ln (e^{\ln 3 + 1})$
   \[ 2 + x = \ln 3 + 1 \]
   \[ x = \ln 3 - 1 \]
   \[ x = 0.1 \]

8) C: $\ln (4e^x) = \ln 4 - \ln e^x = \ln 4 - 6 = -6$
   \[ x = \ln 4 + 6 = 7.4 \]

9) B: $N(t) = 5000 e^{3(5)} = 5000 e^{1.5} = 22,408$

10) D: $50,000 - 5,000e^{3t}$
    \[ 10 = e^{3t} \]
    \[ t = \frac{\ln 10}{3} = 7.67 \text{ hours or nearly 8 hours} \]

11) where are 11 - 15?

Test B

1) C
2) B
3) B

4) D: $\lim_{x \to 5} \frac{(x^2 - 3x - 10)}{x - 5} = \lim_{x \to 5} \frac{x + 2}{(x - 5)} =$
   \[ \lim_{x \to 5} - (x + 2) = -(5 + 2) = -7 \]

5) D

6) C: $\lim_{x \to \infty} \frac{e^{-2x} + 2e^{-x}}{e^{-x}} = \lim_{x \to \infty} \frac{(e^{-x} + 2)}{(e^{-x})} =$
   \[ \lim_{x \to \infty} (e^{-x} + 2) = 2 \]

7) A: $\lim_{\theta \to 0^\circ} \frac{\sin^2 \theta}{\sin (\theta - 1)} = \lim_{\theta \to 0^\circ} \frac{\sin \theta}{\sin \theta - 1} = \lim_{\theta \to 0^\circ} 0 = 0$

8) A: $\lim_{x \to 1} \frac{2 \ln x + 3}{x} = \lim_{x \to 1} 2 \ln x + 3 = 3$

9) A: $\lim_{x \to 4} \frac{1}{x^4 - 4x} = \frac{1}{4^4 - 4} = 1/256 - 2 = 1/254$

10) A: $\lim_{x \to \infty} -\frac{1}{2x} + 7 = 7$ (the first term gets closer and closer to 0)

11) D: Absolute value cannot be negative.

12) A: Solving the equality gives us $X = 48$. Substituting 0 for $X$ in the original inequality gives a true result, so the solution is $X < 48$.

13) C: $\sqrt[3]{B^2} = B^{\frac{2}{3}} = B^{\frac{1}{3}}$

14) B: $\frac{3(1 - \cos^2 \theta)}{\cos (90^\circ - \theta)} = \frac{3 \sin^2 \theta}{\sin \theta} = 3 \sin \theta; 3 \sin 90^\circ = 3$

15) A: $\frac{B^2B^{-3}C^4}{C^{-2}B} = \frac{B^{-1}C^4}{B^{-2}} = \frac{C^2C^4}{B^2} = \frac{C^6}{B^2}$